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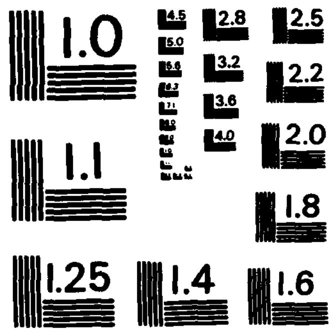

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STATISTICAL EFFECTS OF IMPERFECT INSPECTION SAMPLING:
II. DOUBLE SAMPLING AND LINK SAMPLING

by

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ABSTRACT

This paper, the second in a series of three, utilizes the basic distributions established in the first paper to assess the effect of inspection errors in three types of acceptance procedures: double sampling, link and partial link sampling.

The numbering of sections, equations and tables follows from that of the previous paper in this series (Johnson, Kotz and Rodriguez (JKR) (1985)).

KEYWORDS: Acceptance Sampling Plans; Binomial Distribution; Compound Distribution; Double Sampling, Link Sampling, Hypergeometric Distribution, Inspection Error Tables.

Double Sampling

A double sampling procedure is defined by the values of five parameters:

n_1, n_2 - sizes of first and second stage random samples, respectively

a_1, a_2 - acceptance numbers at first and second stage, respectively

$a_1' + 1$ - rejection number at first-stage.

Letting Z_1, Z_2 denote the numbers of items classified (rightly or wrongly) as 'nonconforming' in the first and second stage samples, respectively, the procedure is as follows; -

(i) Take a random sample of size n, and record the (apparent) number, Z_1, of nonconforming items.

(ii) If $Z_1 \leq a_1$, accept. If $Z_1 > a_1'$, reject.

(iii) If $a_1 < Z_1 \leq a_1'$, takes a further random sample of size n_2, and record the (apparent) number, Z_2, of nonconforming items.

(iv) If $Z_1 + Z_2 < a_2$, accept. If $Z_1 + Z_2 > a_2$, reject.

(Commonly, though not necessarily, $n_2 = 2n_1$ and $a_2 = a_1'$.)

Table 3 gives acceptance probabilities for some of the double sampling procedures defined in Tables III-A, III-B, and III-C of <u>Sampling Procedures and Tables for Inspection by Attributes</u> (1981). Computational details are given in Appendix A. The parameter combinations represented in Table 3 are N = 100,200; D/N = 0.05, 0.10, 0.20; p = 0.75, 0.90, 0.95, 0.98, 1.00; p' = 0, 0.01, 0.02, 0.25 0.10. Figures 3a-c, 4a-c, 5a-c and 6a-c provide graphic representation of these values. In each set of these figures there are presented results from those double sampling schemes corresponding to average quality levels (AQL) of 1.5%, 4% and 10% nonconforming items respectively.

In Table 3 and Figures 3, 4, 5, and 6, the case $p=1$ and $p'=0$ corresponds to perfect inspection. As one might expect, for fixed p' the probability of acceptance increases as p decreases (fewer nonconforming items are correctly classified.) Moreover, the probability of acceptance decreases as p' increases (more conforming items are incorrectly classified.) It is clear from Table 3 and Figures 3-6 that the effect of increasing p' is relatively greater than the effect of decreasing p. Values of p as low as 0.95 do not have a great effect on the acceptance probability, whereas values of p' even as small as 0.01 do have a noticeable effect.

Example. The sampling schemes in Military Standard 105D are often analyzed by computing the values of their operating characteristic (OC) curves at the 5% and 10% points. Table 3 and Figures 3-6 can be used to assess the sensitivity of the OC curve to inspection error.

Suppose, for instance, that the double sampling scheme defined by $n_1=13$, $n_2=13$, $a_1=0$, $a_1'=2$, and $a_2=3$ is selected for AQL=4 at Inspection Level II with a lot size N=100. Furthermore, suppose that p is known to be as low as 0.95 and p' is known to be as high as 0.05. What are the limits for the variation in the probability of acceptance?

From Table 3-6:

D/N x100%	Lower limit for Pr(acceptance)	P(acceptance) if no inspection error	Upper limit for Pr(acceptance)
5%	.7474	.9749	.9787
10%	.4448	.7400	.7712

If lots of size N=200 are used instead, the limits (see Table 3-22) are:

D/N x100%	Lower limit for Pr(acceptance)	P(acceptance) if no inspection error	Upper limit for Pr(acceptance)
5%	.6816	.9708	.9761
10%	.3257	.6639	.7048

Note that for low values of the fraction nonconforming (D/N), the probabilities of acceptance for lot sizes N=100 and N=200 do not differ much.

When D is small, variation in p has little effect on the probability of acceptance, since only the D nonconforming items in the lot are affected by the incorrect classification.

Example. Again consider the sampling scheme defined by $n_1=13$, $n_2=13$, $a_1=0$, $a_1'=2$, and $a_2=3$ selected for AQL=4, Inspection Level II, and lot size N=100. Suppose that p' is known to be approximately 0.02, but no estimate of p is available. What is the lower bound for p if an increase in the probability of acceptance of at most 0.04 is allowed?

Using Table 3-6 and interpolating linearly:

D	Lower bound for p
5	0.75
10	0.94
20	0.94

Link Sampling and Partial Link Sampling

As a cost-saving alternative to double sampling, it has been proposed to use results of routine samples from neighboring lots in the production sequence for the second sample, when needed. Harishchandra and Srivenkataramana (HS) (1982) describe the following 'link sampling' procedure, based on random samples (without replacement) of size n each from a sequence of lots of size N. We will use D_i to denote the actual number of nonconforming items in the i-th lot, Y_i to denote the actual number of nonconforming items in the sample of size n from this lot, Y_i to denote the actual number of nonconforming items in the sample of size n from this lot, and Z_i to denote the number classified as nonconforming in this sample. The link sampling division rules for the i-th lot are:

(a) If $Z_i \leq a_1$ the lot is accepted

(b) If $Z_i > a_2$ the lot is rejected

(c) If $a_1 < Z_i \leq a_2$ and $Z_{i-1} + Z_i + Z_{i+1} \leq a'_2$ the lot is accepted

(d) If $a_1 < Z_i \leq a_2$ and $Z_{i-1} + Z_i + Z_{i+1} > a'_2$ the lot is rejected

The integers a_1, a_2, and a'_2 are chosen to give desired acceptance probabilities for specified proportions (D_i/N) of defectives. Typically $a_2 = a'_2$. In link sampling it is assumed that neighboring lots in the production sequence are of the same quality. Thus D_{i-1}, D_i, and D_{i+1} must not differ greatly. In the binomial sampling case (N infinite) considered by HS (1982), if the proportion defective is constant from lot to lot, the acceptance probabilities for link sampling are identical to those for ordinary double sampling where the same values of a_1, a_2, and a'_2 are used, and where a second sample of size 2n is chosen from the i-th lot. Here, however, we are concerned with finite lot sizes, and a similar result does not hold when $D_{i-1} = D_i = D_{i+1}$ because the convolution $\text{Hypg}(n;D;N) * \text{Hypg}(n;D;N)$ is not the same as Hypg (2n;D;N) (or Hypg (2n;2D;2N)). Before presenting the results for the finite lot size situation, we discuss two alternative forms

of link sampling suggested by HS (1982).

A drawback of the procedure outlined above is that it is necessary to wait for the results of inspection of the $(i+1)$st lot before reaching a decision on the i-th lot when $a_1 < z < +a_2$. To reduce the time needed to reach a decision, HS (1982) propose the two following methods:

(1) In (c) (and (d)) replace $Z_{i-1} + Z_i + Z_{i+1} \leq (>)\ a_2'$ by

$$Z_{i-1} + Z_i \leq (>)\ a_2''$$

or (2) If $a_1 < Z_i \leq a_2$ then take a second sample of size n (not 2n) from the i-th lot, and replace Z_{i+1} (in (a) and (d)) by Z_i', the number of items classified as nonconforming in this second sample.

Method (1) may rely too heavily on the assumption of constant D_i. This difficulty is met by Method (2), which is termed 'partial link sampling' - it saves some sampling effort as compared with regular double sampling, and avoids the need to wait for results of sampling the next $((i+1)$-th) lot. See Appendix B for mathematical details.

Tables 4 and 5 contain values of acceptance probabilities for link sampling and partial link sampling, respectively, for the same values of p and p' as in Table 3, and for a few sampling schemes chosen for illustrative purposes.

Tables 4-1 and 4-2 are comparable with 5-1, and Table 4-3 with 5-2. Table 4-4 can be compared with 5-3, and Tables 4-5, 4-6 and 4-7 with 5-4.

Figures 7a-c give some link sampling acceptance probability distributions. They can be compared with Figure 7d which gives the distributions for double sampling with the same sample sizes and with the same number of nonconforming items (D=20) in the lot under inspection. For this lot size (N=100) the differences are

minor, but for lot size 70 (see Figures 8a and 8b) the discrepancies between double sampling and link sampling with the same (constant) D values become much more marked. (Note that if the second sample is used then 60 out of the 70 items in the lot are examined, so this is a rather extreme case.

Conclusions

The basic distributional results derived in JKR (1985) yield exact, closed form expressions for the acceptance probabilities associated with ordinary double sampling and link sampling procedures for finite lot sizes in the presence of inspection error. Computation of the acceptance probabilities reveals the effects of varying p and p', as well as N, n, and D.

- For a wide range of double sampling schemes, the probability of acceptance is insensitive to p as low as 0.95, but it is sensitive to p' as small as 0.01. A similar conclusion applies to the link sampling schemes considered.

- For double sampling, the effects of changes in p and p' are more pronounced for larger sample sizes.

- For double sampling, the probabilities of acceptance for a given fraction defective (D/N) for lot sizes N=100 and N=200 do not differ greatly.

- For link sampling, the probability of acceptance appears to be sensitive to differences between D_i and the average of D_{i-1} and D_{i+1} (see figs. 7).

Appendix A

Acceptance Probabilities in Double Sampling

The probability of acceptance is

$$\Pr[Z_1 \le a_1] + \Pr[(a_1 < Z_1 \le a_1') \cap (Z_1 + Z_2 \le a_2)] \tag{15}$$

We suppose (as in JKR (1985)) that sampling is without replacement. The probabilities in (15) depend on p (probability of detecting a nonconforming item) and p' (probability of incorrectly describing a conforming item as "nonconforming') as well as the parameters of the sampling procedure, N (the size of the lot) and D (the number of nonconforming items in the lot). The first term in (15) is evaluated by summing (4') (with suffixes '1' for z) over $0 \le z \le a_1$; the second is evaluated by summing (8)' (with $k = 2$) over z_1 and z_2 for which $(a_1 < z_1 \le a_1')$ and $(z_1 + z_2 \le a_2)$.

Appendix B

Acceptance Probabilities in Link Sampling

The Z_i's are mutually independent; Z_i has a distribution of form (1) with D replaced by D_i. The probability of acceptance for the i-th lot is:

$$\Pr[Z_i \le a_1 | D_i] + \sum_{z=a_1+1}^{a_2} \Pr[Z_i = z | D_i] \Pr[Z_{i-1} + Z_{i+1} \le a_2' - z | D_{i-1}, D_{i+1}]$$

$$= \sum_{z_i=0}^{a_1} P(z_i|D_i) + \sum_{z_i=a_1+1}^{a_2} P(z_i|D_i) \sum_{z_{i+1}=0}^{a_2'-z_i} \sum_{z_{i-1}=0}^{a_2'-z_i-z_{i-1}} P(z_{i-1}|D_{i-1}) P(z_{i+1}|D_{i+1})$$

where $P(z_j|D_j) = \Pr[Z_j = z_j]$ (16)

$$= \binom{N}{n}^{-1} \sum_y \binom{D_j}{y}\binom{N-D_j}{y} \sum_{u=0}^{y} \binom{y}{u}\binom{n-y}{z_j-u} p^u p'^{z_j-u} (1-p)^{y-u} (1-p')^{n-y-z_j+u}$$

(cf (2))

The expected number of items inspected in the i-th lot is $n\{1+\Pr[a_1 < Z_i \le a_2']\}$, while with regular double sampling (with $n_1=n$, $n_2=2n$) it is $n\{1+2\Pr[a_1 < Z_i \le a_2']\}$.

Appendix C

Acceptance Probabilities in Partial Link Sampling

The analysis is a bit more complicated than for link sampling because Z_i and Z_i' are not independent (though they would be for sampling with replacement, or for N infinite). The joint distribution of Z_i and Z_i' is symbolically

$$\begin{Bmatrix} Z_i \\ Z_i' \end{Bmatrix} \cap \begin{Bmatrix} \mathrm{Bin}(Y_i, p) * \mathrm{Bin}(n-Y_i, p') \\ \mathrm{Bin}(Y_i', p) * \mathrm{Bin}(n-Y_i', p') \end{Bmatrix} \bigwedge_{Y_i, Y_i'} \text{Mult Hypg}(n,n;D_i;N) \qquad (17)$$

(Y_i' denotes the actual number of nonconforming items in the second sample from the i-th lot, and the joint distribution of Y_i, Y_i' is given by (7) with $k=2$, $n_1=n_2=n$.)

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